Competitive Equilibrium

Econ 3030 Fall 2025

Lecture 19

Outline

1. Competitive (Walrasian) Equilibrium

Decentralized vs. Centralized Economic Systems

- So far, the economy's outcomes have been given 'top-down'.
- Someone, somewhere, somehow, tells all consumers and all firms what to do.
- This is sometimes called a "centralized" economic system.

- Next, we introduce a "decentralized" economic system: individuals are free to choose what they want, given the constraints they face.
- All transactions take place simultaneously in a place called the market.
- There, consumers sell their initial endowment and buy consumption bundles, while firms buy inputs and sell outputs.
- The main notion is that of a competitive (Walrasian) equilibrium.
- It is defined in three pieces: consumers maximize utility, firms maximize profits, and markets clear.

Markets

- Assume there is a maket for each of the L commodities.
  - In these markets, commodities can be bought and sold in perfectly divisible quantities.
- There is a unit of account for evaluating purchases and sales; this may be thought of as money.
- The price of commodity l in terms of the unit of account is denoted by p_l.

- A price vector is $\mathbf{p}$. No price can be negative, and at least one price must be strictly positive: $\mathbf{p} \in \mathbb{R}^L_+$.
- Given prices $\mathbf{p} \in \mathbb{R}^L_+$, the cost of consumption bundle $\mathbf{x} \in \mathbb{R}^L_+$ is

$$\mathbf{p} \cdot \mathbf{x} = \sum_{l=1}^{L} p_l x_l$$

- Given prices $\mathbf{p} \in \mathbb{R}^L_+$, the profits of production vector $\mathbf{y} \in \mathbb{R}^L$ are

$$\mathbf{p} \cdot \mathbf{y} = \sum_{l=1}^{L} p_l y_l$$

  - This can be negative since some y_ls are negative.

Markets and Choices: Firms

- Agents (consumers and firms) make decision while taking prices as given: from their point of view, $\mathbf{p}$ is fixed.

Given a price vector $\mathbf{p} \in \mathbb{R}^L_+$:

- Each firm maximizes profits: chooses an output vector from its supply correspondence:

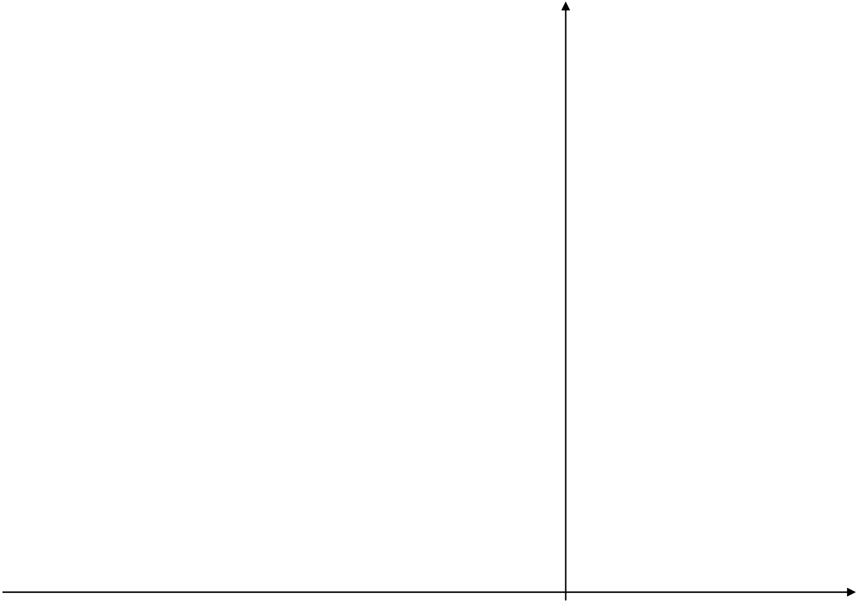
$$\mathbf{y}_j^* \in y_j^*(\mathbf{p})$$

  where

$$y_j^*(\mathbf{p}) = \{\mathbf{y} \in Y_j : \mathbf{p} \cdot \mathbf{y}_j^* \geq \mathbf{p} \cdot \mathbf{y}\}$$

  - In each market, firms can sell output and purchase input

Profit Maximization Picture



Markets and Choices: Consumers

- Agents (consumers and firms) make decision while taking prices as given: from their point of view, $\mathbf{p}$ is fixed.

Given a price vector $\mathbf{p} \in \mathbb{R}^L_+$:

- Each consumer maximizes their preferences: chooses a consumption bundle from their Walrasian demand correspondence:

$$\mathbf{x}_i^* \in x_i^*(\mathbf{p})$$

where

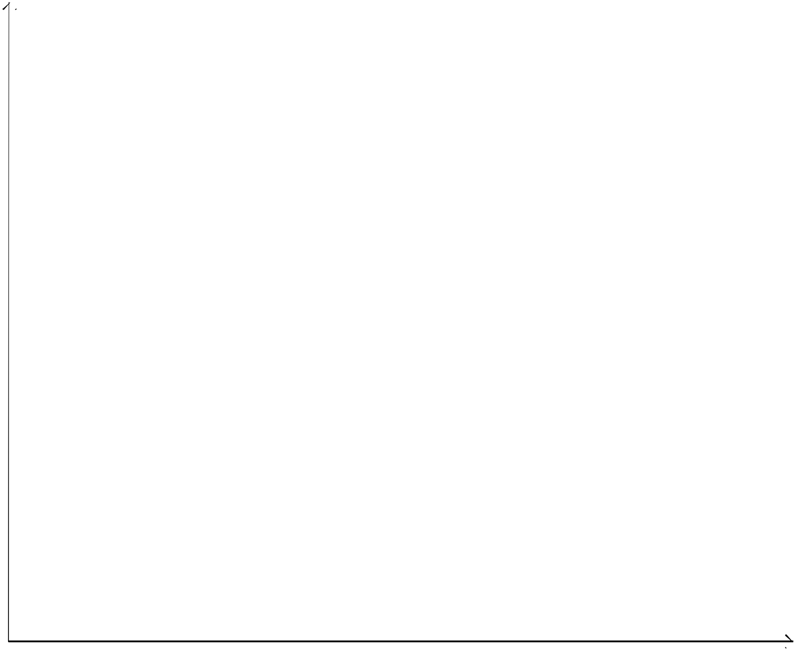
$$x_i^*(\mathbf{p}) = \{\mathbf{x}_i^* \in B_i(\mathbf{p}) : \mathbf{x}_i^* \succsim_i \mathbf{x}_i \quad \text{for each } \mathbf{x}_i \in B_i(\mathbf{p})\}$$

and

$$B_i(\mathbf{p}) = \{\mathbf{x} \in \mathbb{R}^L_+ : \mathbf{p} \cdot \mathbf{x} \leq \mathbf{p} \cdot \boldsymbol{\omega}_i + \sum_{j=1}^{J} \theta_{ij}(\mathbf{p} \cdot \mathbf{y}_j^*) \text{ where } \mathbf{y}_j^* \in y_j^*(\mathbf{p})\}$$

  - In each market, consumers buy goods
  - In each market, consumers sell their endowment
  - Each consumer also derives income from the profits of the firms they own.

Utility Maximization Picture



- Each firm maximizes profits: chooses an output vector from its supply correspondence:
$$\mathbf{y}_j^* \in y_j^*(\mathbf{p}) = \left\{\mathbf{y} \in Y_j : \mathbf{p} \cdot \mathbf{y}_j^* \geq \mathbf{p} \cdot \mathbf{y}\right\}$$
- Each consumer maximizes their preferences: chooses a consumption bundle from their Walrasian demand correspondence:
$$\mathbf{x}_i^* \in x_i^*(\mathbf{p}) = \{\mathbf{x}_i^* \in B_i(\mathbf{p}) : \mathbf{x}_i^* \succsim_i \mathbf{x}_i \quad \text{for each } \mathbf{x}_i \in B_i(\mathbf{p})\}$$
where $B_i(\mathbf{p}) = \{\mathbf{x} \in \mathbb{R}_+^L : \mathbf{p} \cdot \mathbf{x} \leq \mathbf{p} \cdot \boldsymbol{\omega}_i + \sum_{j=1}^J \theta_{ij}(\mathbf{p} \cdot \mathbf{y}_j^*)$ where $\mathbf{y}_j^* \in y_j^*(\mathbf{p})\}$.
- Equilibrium takes care of finding the “right” price vector: the price vector that makes all these independent choices mutually consistent.

Competitive (Walrasian) Equilibrium

Definition

Given an economy $\left\{\{X_i, \succsim_i, \boldsymbol{\omega}_i, \boldsymbol{\theta}_i\}_{i=1}^{I}, \{Y_j\}_{j=1}^{J}\right\}$, a competitive (Walrasian) equilibrium is formed by an allocation $(\mathbf{x}^*, \mathbf{y}^*) \in \mathbb{R}^{L(I+J)}$ *and* a price vector $\mathbf{p}^* \in \mathbb{R}^L_+$ such that:

1. $\mathbf{y}_j^* \in y_j^*(\mathbf{p}^*)$ for each $j = 1, ..., J$
2. $\mathbf{x}_i^* \in x_i^*(\mathbf{p}^*)$ for each $i = 1, ..., I$
3. $\sum_{i=1}^{I} \mathbf{x}_i^* \leq \sum_{i=1}^{I} \boldsymbol{\omega}_i + \sum_{j=1}^{J} \mathbf{y}_j^*$ and if $\sum_{i=1}^{I} x_{li}^* < \sum_{i=1}^{I} \omega_{li} + \sum_{j=1}^{J} y_{lj}^*$ then $p_l^* = 0$

1. Firms' choices are optimal given the equilibrium prices;
2. Consumers' choices are optimal given the equilibrium prices; and
3. Demand cannot exceed supply, and if demand is strictly less then supply the corresponding commodity must be free.

NOTE

Equilibrium prices make $J + I$ optimization problems 'mutually compatible': amazing!

Comments On The Definition of Equilibrium I

A competitive equilibrium is a $(\mathbf{x}^*, \mathbf{y}^*) \in \mathbb{R}^{L(I+J)}$ and a $\mathbf{p}^* \in \mathbb{R}^L_+$ s. t.:

1. $\mathbf{y}_j^* \in y_j^*(\mathbf{p}^*)$ for each $j = 1, ..., J$
2. $\mathbf{x}_i^* \in x_i^*(\mathbf{p}^*)$ for each $i = 1, ..., I$
3. $\sum_{i=1}^{I} \mathbf{x}_i^* \leq \sum_{i=1}^{I} \boldsymbol{\omega}_i + \sum_{j=1}^{J} \mathbf{y}_j^*$ and if $\sum_{i=1}^{I} x_{li}^* < \sum_{i=1}^{I} \omega_{li} + \sum_{j=1}^{J} y_{lj}^*$ then $p_l^* = 0$

Remark

If $(\mathbf{x}^*, \mathbf{y}^*)$ and $\mathbf{p}^*$ form a Walrasian equilibrium, so do $(\mathbf{x}^*, \mathbf{y}^*)$ and $\alpha\mathbf{p}^*$.

- This follows because $y_j^*(\alpha\mathbf{p}^*) = y_j^*(\mathbf{p}^*)$ and $x_i^*(\alpha\mathbf{p}^*) = x_i^*(\mathbf{p}^*)$, and the third condition does not change.
  - Since supply and demand are homogeneous of degree zero, only the direction of an equilibrium price vector matters, not its size (only relative prices matter).
- Thus one can "normalize" prices in different ways: typical choices are to assume one of them equals 1 or to assume that their sum equals 1.

Comments On The Definition of Equilibrium II

A competitive equilibrium is a $(\mathbf{x}^*, \mathbf{y}^*) \in \mathbb{R}^{L(I+J)}$ and a $\mathbf{p}^* \in \mathbb{R}^L_+$ s. t.:

1. $\mathbf{y}_j^* \in y_j^*(\mathbf{p}^*)$ for each $j = 1, ..., J$
2. $\mathbf{x}_i^* \in x_i^*(\mathbf{p}^*)$ for each $i = 1, ..., I$
3. $\sum_{i=1}^{I} \mathbf{x}_i^* \leq \sum_{i=1}^{I} \boldsymbol{\omega}_i + \sum_{j=1}^{J} \mathbf{y}_j^*$ and if $\sum_{i=1}^{I} x_{li}^* < \sum_{i=1}^{I} \omega_{li} + \sum_{j=1}^{J} y_{lj}^*$ then $p_l^* = 0$

- The zero-price condition, although redundant under some restrictions on preferences, is sometimes important because some commodities may not be desirable to some consumers or firms.
- In that case, equilibrium requires the price of those commodities to be zero.
- However, all kind of strange things can happen when the price of something is zero...
  - Demand for that something (by consumers and/or firms) could equal infinity, for example;
  - in this case there is no solution to the optimization problems, thus preventing the existence of a competitive equilibrium.

Comments On The Definition of Equilibrium III

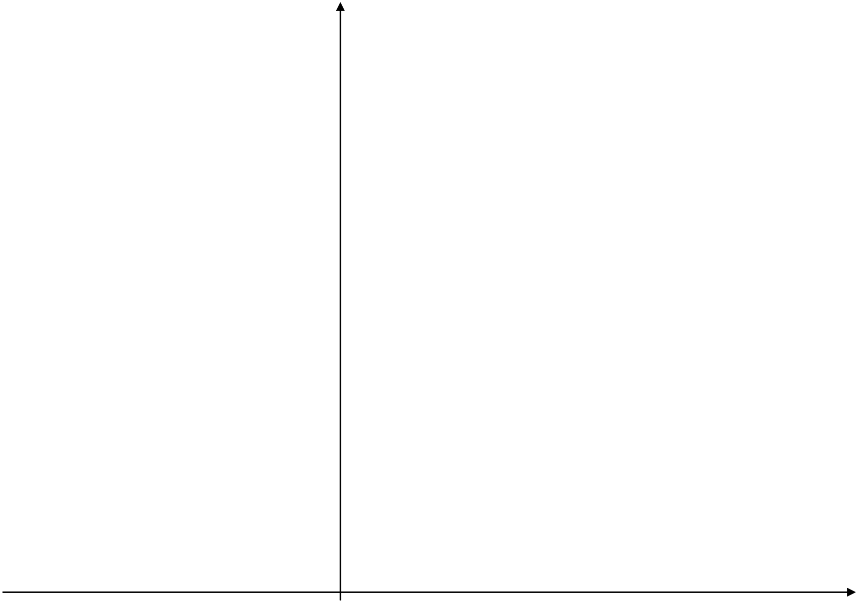
A competitive equilibrium is a $(\mathbf{x}^*, \mathbf{y}^*) \in \mathbb{R}^{L(I+J)}$ and a $\mathbf{p}^* \in \mathbb{R}^L_+$ s. t.:

1. $\mathbf{y}_j^* \in y_j^*(\mathbf{p}^*)$ for each $j = 1, ..., J$
2. $\mathbf{x}_i^* \in x_i^*(\mathbf{p}^*)$ for each $i = 1, ..., I$
3. $\sum_{i=1}^{I} \mathbf{x}_i^* \leq \sum_{i=1}^{I} \boldsymbol{\omega}_i + \sum_{j=1}^{J} \mathbf{y}_j^*$ and if $\sum_{i=1}^{I} x_{li}^* < \sum_{i=1}^{I} \omega_{li} + \sum_{j=1}^{J} y_{lj}^*$ then $p_l^* = 0$

- There is some magic that is important to understand: prices perform a fantastic role in that they make all the individual decisions mutually compatible.
- When prices are not the equilibrium prices, everyone still optimizes but demand is different from supply: the solutons to the optimization problems are mutually incompatible.
- Where do these magic prices come from? The model does not say.
  - Adam Smith talks about the "invisible hand", but that is not math.

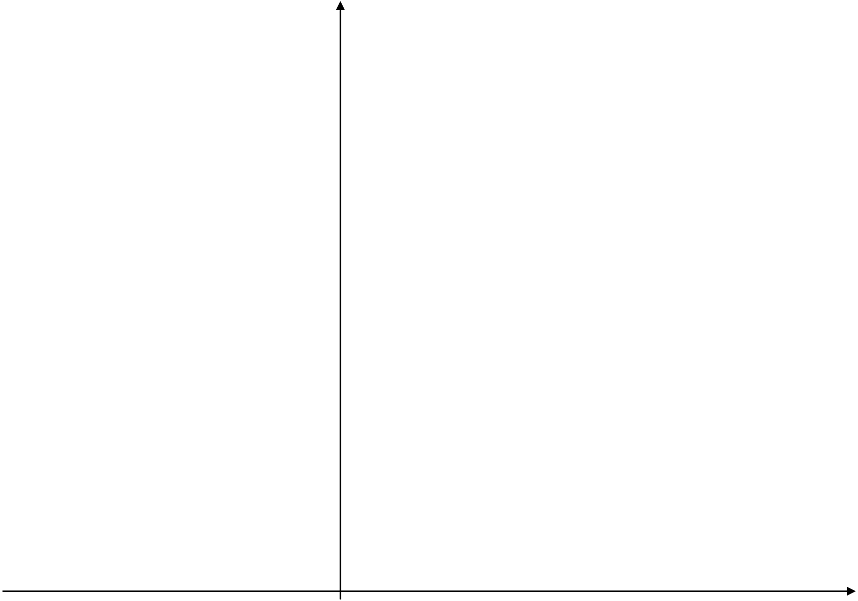
Competitive (Walrasian) Equilibrium: Examples

Representative Agent: Equilibrium vs. non Equilibrium



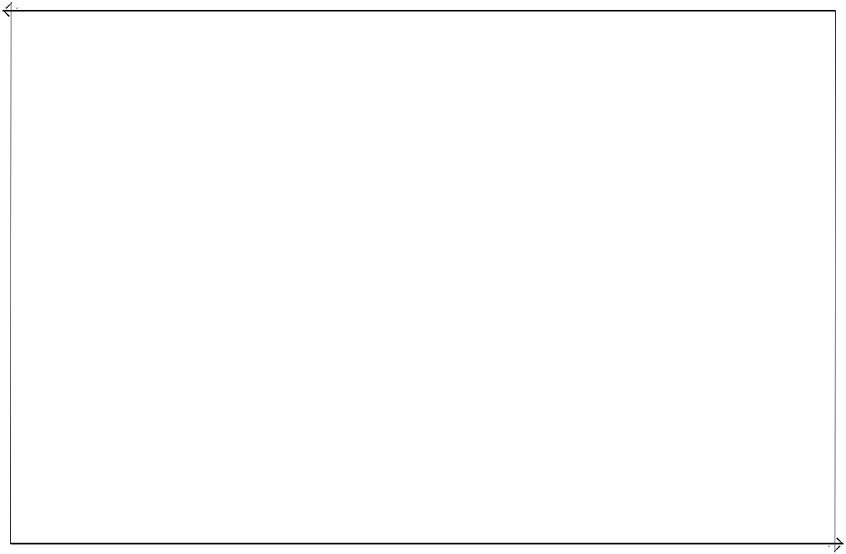
Competitive (Walrasian) Equilibrium: Examples

Representative Agent: Equilibrium vs. non Equilibrium



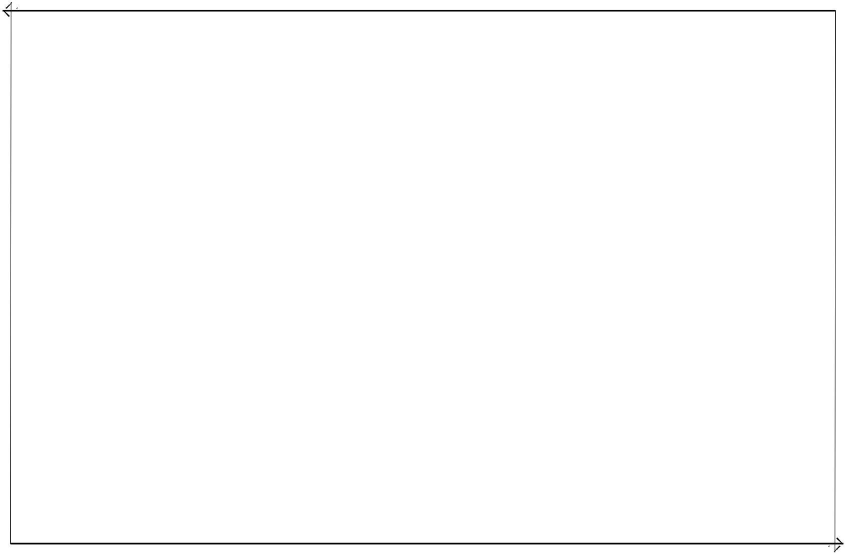
Competitive (Walrasian) Equilibrium: Examples

Exchange Economy: Equilibrium vs. non Equilibrium



Competitive (Walrasian) Equilibrium: Examples

Exchange Economy: Equilibrium vs. non Equilibrium



Calculating Competitive Equilibria

- How do we find a competitive equilibirum? In three easy (?!?) steps.

1. Get $y_j^*(\mathbf{p})$ from the J firms maximization problems.
2. Get $x_i^*(\mathbf{p})$ from the I consumers maximization problems.
3. Find the $\mathbf{p}^*$ such that the L inequalities $\sum_{i=1}^{I} \mathbf{x}_i^* \leq \sum_{i=1}^{I} \boldsymbol{\omega}_i + \sum_{j=1}^{J} \mathbf{y}_j^*$ are all satisfied (in most cases these will be equalities).

- The last step (if given by equalities) asks you to solve a system of L equations (one per good) in L unknowns (the prices of each good).
- But we know that demand and supply are homogeneous of degree zero, so only relative prices can be found.
- This means we only have $L-1$ unknowns and L equations... not good.
- ... Unless one of the L equations is redundant: homework.

Next Class

- Comparison of competitive equilibria and Pareto optimal allocations:
- First Fundamental Theorem of Welfare Economics: any competitive equilibrium is Pareto efficient.
- Proof and discussion of the First Welfare Theorem.
- Second Fundamental Theorem of Welfare Economics: any Pareto optimal allocation can be made into a (special) competitive equilibrium.